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NUMERICAL ANALYSIS OF SANDWICH PANELS SUBJECTED TO TORSION

Zbigniew Pozorski

Institute of Structural Engineering, Poznan University of Technology
Poznan, Poland
zbigniew.pozorski@put.poznan.pl

Abstract. The chapter presents the problem of sandwich panel torsion. Theoretical solutions for beams (1-D elements) and results of numerical analyzes are presented. Two classes (2D and 3D) of numerical models are considered, and the results are compared with the theoretical results. The comparison mainly concerns the stresses in the facings and the core of sandwich structure. The biggest challenge was finding the appropriate boundary conditions (both support and load) for the different model classes. Once again it turned out that the question of boundary conditions is crucial to solving the problem, and that the class of these conditions should always correspond to the class of the model.

Keywords: *sandwich panels, torsion, numerical simulations, Finite Element Method*

1. Introduction

The problem of torsion of sandwich panels with a shear deformable core is known in the literature. We can distinguish at least three theoretical approaches to this issue [1-3]. It is also worth noting that, in the analysis of torsion, it is important to distinguish the St. Venant torsion and the warping (Vlasov) torsion. The problem of torsion was also analyzed experimentally and numerically, even in the papers [4-6].

In most typical cases, which concern the sandwich panels and take place in civil engineering, the effect of torsion is rightly ignored. Even if there are some load eccentricities, the torsion and the induced forces are very small (in practice negligible). For some time, with the tendency to enlarge the thickness of the core, this situation is changing. An even more drastic example, in which torsion is important, is the installation of an additional façade layer to the existing walls made of sandwich panels. The heavier and more distant the additional layer, the larger the torsion of the sandwich panel to which it is mounted.

In this paper, the influence of load and support boundary conditions on the internal forces and stresses in the sandwich structure subjected to torsion is analyzed. At the beginning of the paper, based on [7], theoretical solutions for basic static beams (1-D elements) will be presented. A detailed derivation of the function of the rotational angle will be presented. Then, numerical models will be discussed (2-D and 3-D). The analyzes started with the schemes closest to the theoretically ideal conditions. Subsequently, the conditions to reflect the actual conditions were gradually sought. Although the load and support boundary conditions of the wall panels seem to be fairly straightforward, in fact they are very difficult to reflect in a simple theoretical model. For this reason, the decision was made to perform the appropriate numerical analysis.

2. Torsion of a sandwich beam element

2.1. Differential equation

As mentioned in the introduction, the torsion mainly relates to sandwich panels that are mounted horizontally. The main cause of the torsion is the application of additional load on the eccentric. This situation is illustrated in Figure 1, which shows the cross-section of the element and the load F acting on the eccentric a. It should be clarified that in the case of the beam and plate model, the load and the supports refer to the axis (surface) representing centers of the gravity of the element cross-sections. In the case of the 3-D elements, the thickness of the element appears explicitly. Therefore, the support conditions need not be (and are not usually) related to the center (gravity) surface of the element. Thus 3-D elements, by definition, belong to a completely different class. On the one hand, they guarantee effects that are not taken into account in simpler models, but on the other hand, the results obtained for 3-D models are much more difficult for engineering interpretation.

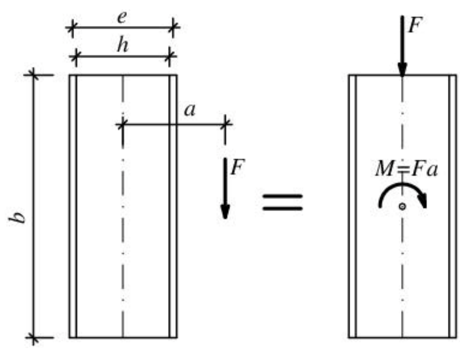


Fig. 1. The cross-section of horizontal sandwich panel with additional eccentric loading inducing torsion of the element

When analyzing a difficult problem (which is certainly a torsion), it is worthwhile to start with the simplest model, which is the beam. The basic static systems of beams subjected to torsion are shown in Figure 2. In schemes 2a and 2b, the beam at both ends has a fork support, i.e., for which the angle of rotation of the cross-section is blocked, but is the freedom of warping (deplanation) of the cross-section. The condition of freedom of deplanation is equivalent to the fact that the second derivative of the angle of rotation is equal to zero (the bimoment equals zero). In the case of scheme 2c, at the left end, there is a fixation that corresponds to the condition of the zero value of the angle of rotation, as well as its first derivative. At the free (right) end of this beam, we have the freedom of warping (deplanation), so the second derivative of the angle of rotation equals zero.

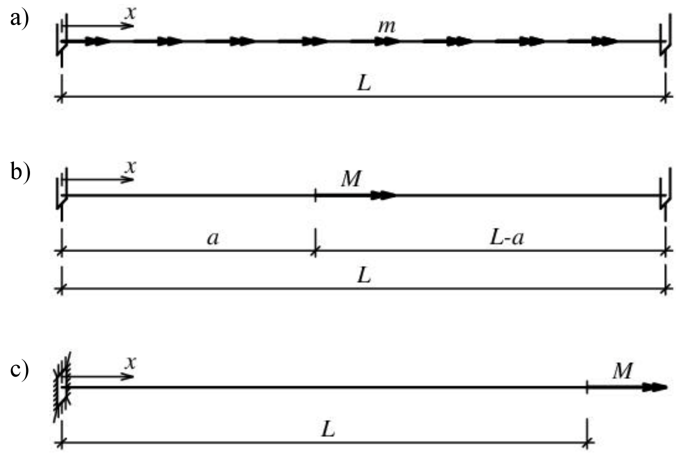


Fig. 2. The examples of single beams subjected to torsion: a) end fork supports and a continuous torsional moment, b) end fork supports and a single torsional moment, c) a cantilever beam subjected to a single torsional moment

The total torsion moment M is decomposed into two effects: the relatively well-known free (St. Venant's) torsion moment M_t and the warping (Vlasov) torsion moment M_w:

$$M(x) = M_t(x) + M_w(x), \tag{1}$$

where x is the coordinate of the point on the beam. Each of these effects can be expressed using the appropriate function of the angle of cross-section rotation φ:

$$M(x) = GI_t\varphi'(x) - EI_w\varphi'''(x), \tag{2}$$

where E, G, I_w and I_t denote modulus of elasticity, shear modulus, second order sectorial moment and torsion constant, respectively (GI_t is the torsion stiffness, EI_w is the warping stiffness). Introducing the constant

$$\lambda^2 = \frac{GI_t}{EI_w}, \quad (\lambda \neq 0), \tag{3}$$

the following heterogeneous differential equation of the variable φ is obtained:

$$\varphi'''(x) - \lambda^2\,\varphi'(x) = -\frac{M(x)}{EI_w}. \tag{4}$$

Knowing the external load, the function of torsion moment $M(x)$ can be found. Solving the equation (4), we obtain the function of the angle of rotation φ and the functions of internal forces (moments M_t, M_w and a bimoment B):

$$M_t(x) = GI_t\varphi'(x), \tag{5}$$

$$B(x) = -EI_w\varphi''(x), \tag{6}$$

$$M_w(x) = -EI_w\varphi'''(x). \tag{7}$$

Knowing all the internal forces, one can determine the distribution of stresses. In the case of sandwich panels (in fact - in sandwich beams), the equation derived in [8] are usually used. The moment M_t induces shear stresses in facings $\tau_{F,t}$ and in the core of sandwich panel $\tau_{C,t}$:

$$\tau_{F,t} = \frac{\cosh(kb/2) - \cosh(ky)}{\cosh(kb/2) - \dfrac{\sinh(kb/2)}{kb/2}} \cdot \frac{M_t}{2hbt_F}, \tag{8}$$

$$\tau_{C,t} = \frac{k\sinh(ky)}{\cosh(kb/2) - \dfrac{\sinh(kb/2)}{kb/2}} \cdot \frac{M_t}{2hb}, \tag{9}$$

where b and h denote the width and depth of the core of the sandwich element and t_F is the thickness of the respective facing (thickness of the upper facing t_{F1} or of the lower facing t_{F2}). The variable y is the coordinate of the cross-section point (axis y is parallel to the facings, extreme values are equal to $\pm b/2$). The constant k is:

$$k^2 = \frac{G_C}{G_F} \cdot \frac{t_{F1} + t_{F2}}{ht_{F1}t_{F2}}, \quad (k \neq 0). \tag{10}$$

The shear modulus G_C and G_F are parameters that correspond to the core and facing material.

The moment M_w induces shear stresses $\tau_{F,w}$ and normal stresses $\sigma_{F,w}$ in facings [7]:

$$\tau_{F,w} = \frac{M_w S_w}{I_w t_F}, \tag{11}$$

$$\sigma_{F,w} = \frac{B}{I_w}\omega, \tag{12}$$

where ω is the sectorial coordinate and S_w is the first order sectorial moment. If both facings have the same thickness, then $\omega_{\max} = be/4$, $S_w = b^2et_F/16$ and $I_w = b^3e^2t_F/24$ (e is the distance between centroids of the facings).

2.2. Solution of the differential equation

In this section, the solution of the differential equation (4) is presented. Three cases of boundary conditions illustrated in Figure 2 are considered. The general integral of (4) has the form:

$$\varphi_0(x) = C_1 \sinh \lambda x + C_2 \cosh \lambda x + C_3, \tag{13}$$

where C_1, C_2 and C_3 are constants. Note that in the case illustrated in Figure 2b, two different solutions (two different sets of constants) are required for two different parts of the beam (on the left and on the right from the applied concentrated moment). The functions of torsion moment $M(x)$ depend on the form of loading, but can be easily found, and the particular integrals φ_1 have the form of polynomials:

a) a single beam with end fork supports, subjected to a continuous torsional moment m (Fig. 2a):

$$M(x) = \frac{mL}{2} - mx, \tag{14}$$

$$\varphi_1(x) = -\frac{m}{2\lambda^2 EI_w} x^2 + \frac{mL}{2\lambda^2 EI_w} x, \tag{15}$$

b) a single beam with end fork supports, subjected to a single torsional moment M (Fig. 2b):

$$M(x) = \begin{cases} \frac{L-a}{L} M & \text{if } \; 0 \le x \le a \\ -\frac{a}{L} M & \text{if } \; a < x \le L \end{cases}, \tag{16}$$

$$\varphi_1(x) = \begin{cases} \frac{L-a}{L} \cdot \frac{M}{\lambda^2 EI_w} x & \text{if } \; 0 \le x \le a \\ -\frac{a}{L} \cdot \frac{M}{\lambda^2 EI_w} x & \text{if } \; a < x \le L \end{cases}, \tag{17}$$

c) a cantilever beam subjected to a single torsional moment M (Fig. 2c):

$$M(x) = M, \tag{18}$$

$$\varphi_1(x) = \frac{M}{\lambda^2 EI_w} x. \tag{19}$$

To solve the problem (4) with $\varphi(x) = \varphi_0(x) + \varphi_1(x)$ and (14-15), (16-17) or (18-19), the appropriate boundary conditions should be taken into account:

a) a single beam subjected to a continuous torsional moment m (Fig. 2a):

$$\varphi(0) = 0, \;\; \varphi''(0) = 0, \tag{20}$$

$$\varphi(L) = 0, \;\; \varphi''(L) = 0, \tag{21}$$

b) a single beam subjected to a single torsional moment M (Fig. 2b):

$$\varphi(0) = 0, \quad \varphi''(0) = 0, \tag{22}$$

$$\varphi(L) = 0, \quad \varphi''(L) = 0, \tag{23}$$

and conditions of continuity (subscripts l and r denote left and right-hand side from the point of concentrated moment application)

$$\varphi_l(a) = \varphi_r(a), \quad \varphi_l'(a) = \varphi_r'(a), \tag{24}$$

c) a cantilever beam subjected to a single torsional moment M (Fig. 2c):

$$\varphi(0) = 0, \quad \varphi'(0) = 0, \tag{25}$$

$$\varphi''(L) = 0. \tag{26}$$

In the first case, we have three unknown C constants and four boundary conditions, but if we meet three conditions, then the fourth is fulfilled automatically. In the second case, we have six unknown constants (three in each part of the beam) and six boundary conditions. In the third case, we have three unknowns and three conditions. Taking into account equations (20-21), (22-24) or (25-26) leads to the final solutions:

a) a single subjected to a continuous torsional moment m (Fig. 2a)

$$\varphi(x) = \frac{m}{\lambda^4 EI_w} \cdot \frac{1-\cosh\lambda L}{\sinh\lambda L} \cdot \sinh\lambda x + \frac{m}{\lambda^4 EI_w} \cdot \cosh\lambda L - \frac{m}{\lambda^4 EI_w} - \frac{m}{2\lambda^2 EI_w} x^2 + \frac{mL}{2\lambda^2 EI_w}, \tag{27}$$

b) a single beam subjected to a single torsional moment M (Fig. 2b)

$$\varphi_l(x) = \frac{M}{\lambda^3 EI_w}\left[\frac{\cosh\lambda L}{\sinh\lambda L} \cdot \sinh\lambda a - \cosh\lambda a\right] \cdot \sinh\lambda x + \frac{L-a}{L} \cdot \frac{M}{\lambda^2 EI_w} x, \tag{28}$$

$$\varphi_r(x) = \frac{M}{\lambda^3 EI_w}\left[\frac{\cosh\lambda L}{\sinh\lambda L} \cdot \sinh\lambda a\right] \cdot \sinh\lambda x - \frac{M \cdot \sinh\lambda a}{\lambda^3 EI_w} \cdot \cosh\lambda x + \frac{aM}{\lambda^2 EI_w} - \frac{a}{L} \cdot \frac{M}{\lambda^2 EI_w} x, \tag{29}$$

c) a cantilever beam subjected to a single torsional moment M (Fig. 2c)

$$\varphi(x) = -\frac{M}{\lambda^3 EI_w} \cdot \sinh\lambda x + \frac{M}{\lambda^3 EI_w} \cdot \frac{\sinh\lambda L}{\cosh\lambda L} \cdot \cosh\lambda x - \frac{M}{\lambda^3 EI_w} \cdot \frac{\sinh\lambda L}{\cosh\lambda L} + \frac{M}{\lambda^2 EI_w} x. \tag{30}$$

Knowing the function $\varphi(x)$ we can find internal forces (5-7) and stresses (8-9, 11-12).

2.3. Example

To illustrate the equations discussed above, the solution of the beam with a span $L = 4.0$ m and a width $b = 1$ m is presented in Figure 3. The thickness of the facings is $t_F = 0.5$ mm and the core thickness is $h = 0.99$ mm (total depth $D = 100$ mm). The material of facings is steel ($E_F = 210$ GPa, $G_F = 81$ GPa), and the shear modulus of the core was assumed as $G_C = 3.5$ MPa. At both ends of the beam there are fork supports. The concentrated torsional moment $M = 0.333$ kNm acts in the middle of the beam (Fig. 2b). Using equations (28-29) and (5-7), the functions of internal forces were found. The forces are presented in Figure 3. Then, using equations (8-12), the extreme values of the stresses are obtained:

$$\tau_{F,t}(x = 0, y = 0) = 2.26\ \text{MPa}, \tag{31}$$

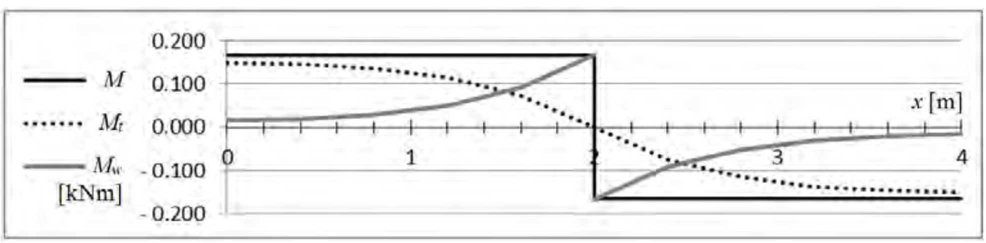
$$\tau_{C,t}(x = 0, y = b/2) = 4.68\ \text{kPa}, \tag{32}$$

$$\tau_{F,w}(x = 0) = 0.50\ \text{MPa}, \quad \tau_{F,w}(x = L/2) = 5.03\ \text{MPa}, \tag{33}$$

$$\sigma_{F,w}(x = L/2, y = b/2) = 13.30\ \text{MPa}. \tag{34}$$

The extreme angle of the cross-section rotation is $\varphi = 0.002277$ [–] (eq. (28)), which means vertical displacements in the center of the span of the panel (at its edges) equal to 1.1385 mm.

a)



b)

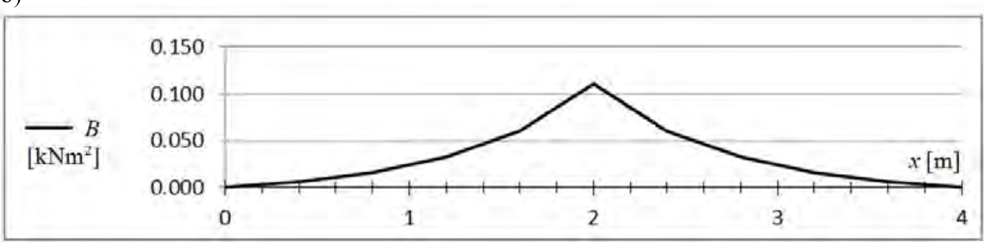


Fig. 3. Functions of internal forces for the one-span beam ($L = 4$ m) with fork support at both ends, subjected to concentrated torsional moment $M = 0.333$ kNm: a) torsional moments, b) bimoment

3. Torsion of a sandwich plate element

All numerical models have been prepared in the Abaqus system. The 2-D model was created applying three-layered shell elements. Thickness of layers and material parameters of each layer were specified respectively (E_F = 210 GPa, G_F = 81 GPa, G_C = 3.5 MPa). For example, a sandwich with total depth D = 100 mm has a steel facing with a thickness of 0.5 mm and the polyurethane core with a thickness of 99.0 mm. The model was discretized using four-node, general-purpose, finite membrane strains, conventional shell S4 elements. The mesh size was constant and equal to 0.02 m. The shell's midsurface was defined as the reference surface containing the element's nodes. The support conditions and the load conditions were specified for the nodes of the model. The concentrated torsional moment M = 0.333 kNm was applied in center of the plate.

Although one may wonder if there is any better way to define the torsional moment, even greater attention should be paid to the boundary conditions. After several attempts, boundary conditions were established as in Figure 4. These conditions are as close as possible to the conditions specified for the beam. At all points of supports (except for point A) there is a freedom of movement along the x-axis.

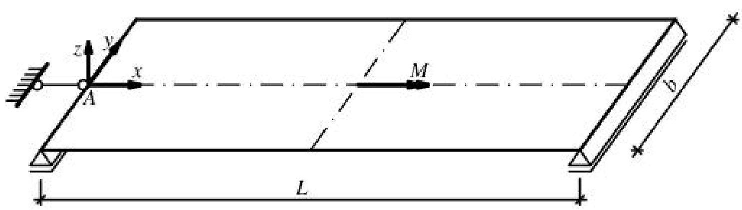


Fig. 4. Definition of boundary conditions for the one-span plate subjected to concentrated torsional moment

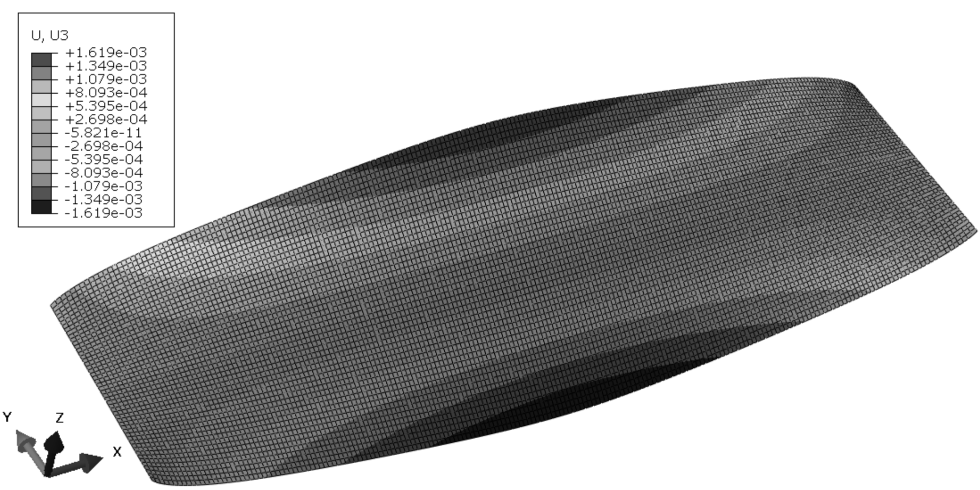


Fig. 5. Vertical displacement of a sandwich plate (2-D) loaded by a concentrated torsional moment applied at the center of the plate

Figure 5 shows the vertical displacements of the plate, which are the effect of the twisting of the element. Extreme displacement is equal to 1.619 mm. Other characteristic results of the numerical solution are summarized in Table 1. However, the values of extreme stresses near the concentrated moment are omitted. It turns out that the local load (concentrated moment) causes unreal stress concentrations. In such cases, the numerical result (locally) depends on the size of the FEM mesh. Fortunately, in fact, we never have to deal with the load concentrated at a point.

4. Torsion of a sandwich 3-D structure

The 3-D model consists of two thin steel facings and a thick but flexible core. The dimensions and material parameters of the layers are the same as in the case of the beam and the plate structure. The 3-D structure has the total length 4.06 m and is supported by two rigid supporting plates, each 0.06 m wide. As a result, the span between the axes of the supports is 4.00 m. The supporting plates are tied with the lower facing of the sandwich structure. The tie connection means that all displacements and rotations of the supporting plate and the sandwich panel (its bottom facing) are identical. The reference points (*A* and *B*) were defined on the support plates, where support conditions were defined (Fig. 6). The following condition was assumed for both supports:

$$u_z = 0, \tag{35}$$

where u_z denotes vertical displacement. In addition, at the point *A* (in the middle of the left support) the horizontal displacement of the lower facing is equal to zero:

$$u_x(A) = 0, \quad u_y(A) = 0, \tag{36}$$

and at the point *B* (in the middle of the right support) one of the horizontal displacements is limited:

$$u_x(B) = 0. \tag{37}$$

All other displacements and rotations are free. These types of supports correspond to the supports defined for the plate and the beam model.

A slightly different issue is the method of loading the sandwich panel. Various cases were considered, including a concentrated force or load spreading in a certain area. The important question is whether, besides torsion, the applied load causes yet another force. The action of the force applied to the external facing induces additional bending of the facing (in-plane) as well as shearing of the core resulting from the transmission of the load towards the supports. This transmission is one of the most interesting and, to date, poorly recognized phenomena. The problem of in-plane bending is also not entirely clear and raises a lot of doubts. First of all, it is a question of uniformity of the load distribution on both facings. Second, there is some uncertainty about the proportions of the "sandwich beam" dimensions. A 3-D layered structure is often treated simply as a beam, although in the case in question, it is closer to the slab structure. Finally, the effect of the local instability of thin facings should also be taken into account. So far, in engineering practice, the impact of in-plane bending of the facings (including the influence of this bending on local instability of facings) was simply neglected.

Finally, it was assumed that for the comparison of results with other models, the most appropriate would be the uniform, tangential to the facing load, distributed over the middle band of both facings (Fig. 6). The width of the band is $w = 0.06$ m. Of course, the direction of the load on the lower facing is opposite to the direction of the load on the upper facing. The load value is $q = 55.83$ kN/m^2, which corresponds to resulting torsional moment $M = 0.333$ kNm. The obtained numerical solution was compared with the results of other models (Table 1), and then commented on and illustrated (Figs. 7 and 8).

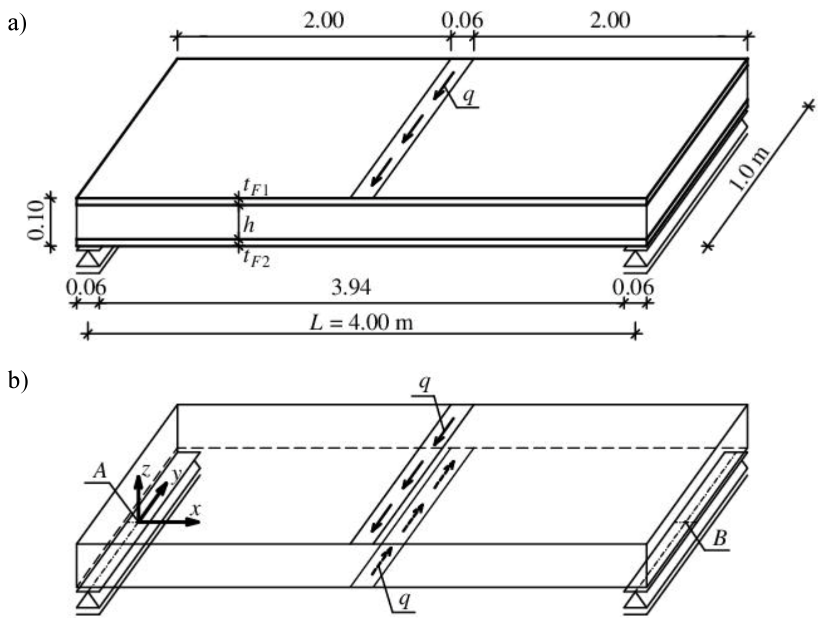


Fig. 6. The 3-D sandwich structure: a) geometry, b) boundary conditions

5. Comparison of the results

The results obtained for three different types of models are presented in Table 1. When comparing these results, it is worth remembering that the class of these models is different, and it happens that the observable effects in one of the models are completely neglected in another. The most primitive is the beam model. On the other hand, it is attractive because it is easy to interpret. The most complex is the 3-D model. It includes, inter alia, the effect of core compression and that the support is offset relative to the center plane of the structure. The 3-D model also offers much greater possibilities in load definition.

The results shown in Table 1 require some explanation. First of all, the 2-D and 3-D models have similar vertical displacements, but these displacements differ from the theoretical value (1-D model). It follows that in the 2-D and 3-D models the condition of rotation restriction (mainly on the support) is much less restrictive than in the 1-D model. Higher class models do not have a rigid contour of the cross-section, which is the basic assumption for the 1-D model.

Comparing the tangential stresses in facings, it can be seen that the results differ for each model. It is interesting that at the support, results close to zero were obtained for both numerical models. This demonstrates the lack of full compatibility between the support conditions for the different models. This is already noticeable when comparing vertical displacements.

Table 1

The comparison of the extreme values obtained for different models of sandwich structure subjected to concentrated torsional moment

Model	1-D model	2-D model	3-D model
Vertical displacement u_z [mm]	1.1385	1.619	1.614
Shear stress in facing for $x = 0$ $\tau_F = \tau_{xy}$ [MPa]	2.26 + 0.50 = 2.76	0.13	0.34
Shear stress in facing for $x = L/2$ $\tau_F = \tau_{xy}$ [MPa]	0.00 + 5.03 = 5.03	8.13	3.77
Shear stress in the core for $x = 0$, $y = b/2$ $\tau_C = \tau_{xz}$ [kPa]	4.68	3.66	15.01
Normal stress in the facing for $x = L/2$, $y = b/2$ $\sigma_F = \sigma_{xx}$ [MPa]	13.30	11.16	20.50

Comparison of the stresses in the middle of the span of elements looks better. In each of the models, results of the same order were obtained, although the value given for the 2-D model (8.13 MPa) is not an extreme value. The extreme stress (85.23 MPa!) occurs in the node in which the concentrated moment is applied. The value 8.13 MPa occurs outside the stress concentration area (0.12 m from the point of concentrated moment application). This illustrates some of the consequences of using concentrated interactions in higher-class models.

The next issue is the shear stress in the core. The highest values were obtained for the 3-D model. This is most reasonable considering that this model also considers the core compression, and the value given in the table (15.01 kPa) is a local value (Fig. 7). Just averaging the stress on the thickness gives a slightly lower value (in the order of 10 kPa). The stress results for the core in the 3-D model are generally much richer than in any other model, because at each point a full tensor of stresses is obtained.

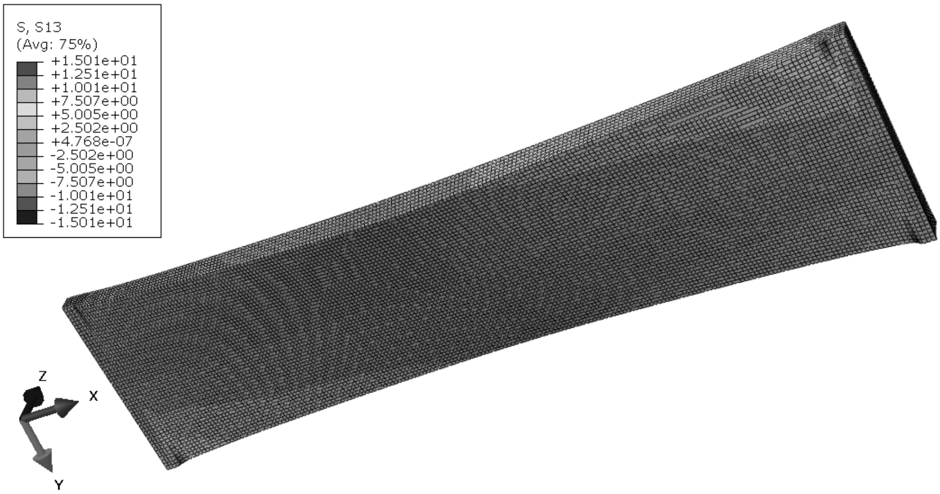


Fig. 7. Shear stress τ_{xz} in the core of the 3-D sandwich structure subjected to torsional moment localized in the midspan; local concentrations are visible at the corners of the panel, close to the supports

The last issue is the normal stress in the facings (last row in Table 1). The 3-D model shows slightly higher stresses, however a more accurate analysis of the results (Fig. 8) indicates that this value is very local. Extreme stresses occur on the edge of the panel. In neighboring nodes, the stresses are of the same order as in the 1-D and 2-D models. The concentration of stresses results, among other things, from the method of loading, although the concentration is still low in comparison to other load schemes (e.g. loads applied only to the edges of facings).

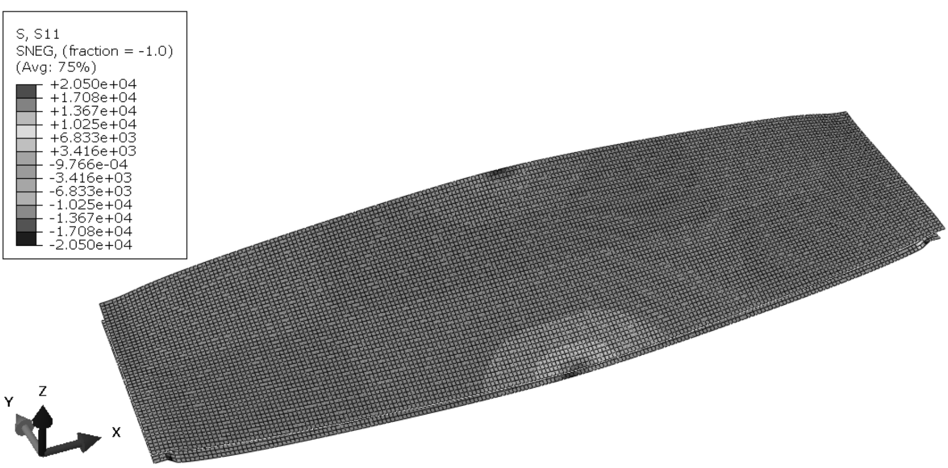


Fig. 8. Normal stress σ_{xx} in the facings of the 3-D sandwich structure subjected to torsional moment localized in the midspan; local concentrations are visible at the edges of the panel

6. Conclusions

This paper presents various aspects of the problem of torsion of sandwich panels. Three different classes of models (beam, plate and 3-D structure) were created and compared. For the simplest beam model the differential equilibrium equation was presented and solved for various support and load conditions. The 2-D and 3-D models were created using the finite element method (Abaqus program).

The work presents an example of a twisted beam for which internal forces have been determined. The example illustrates the importance of the components of the torsional moment. The total moment M consists of St. Venant moment M_t and the warping (Vlasov) moment M_w. The contribution of each of these components is dependent, inter alia, on the level of confinement of the cross-section.

The comparison of the results obtained for different models leads to interesting observations. First of all, the theoretical model based on [1] showed the highest torsional stiffness (the smallest angle of rotation), and the difference from the higher-order numerical models was significant (about 40%). On the other hand, in the case of the 1-D model, higher shear stresses in the facings (at the supports) were obtained. The shear stresses in the core and the normal stresses in the facings in each model were of similar order, although the difficulty in interpreting the numerical results was that there were significant concentrations of stresses at the point of concentrated moments or support reactions. A striking example is the 2-D model and the stresses occurring at the point at which the concentrated torsional moment was applied.

Due to the current tendencies in the technical solutions of sandwich panels, namely the increasingly thicker cores and the application of additional façade layers, it can be assumed that the issue of twisting of laminated elements will arouse more and more attention.

The problem is also extremely interesting because the actual support and load conditions of the panels are complex. This is a problem which, on the one hand, has a solid theoretical basis but also requires further research using modern modeling tools.

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