2D PLANE STRESS MODEL WITH COMPOSITE FIXED-MEMORY FRACTIONAL OPERATORS

Dominika Kustal, Anna Nowak, Tomasz Błaszczyk

Department of Mathematics, Czestochowa University of Technology,
Czestochowa, Poland
dominika.kustal@pcz.pl, anna.nowak@pcz.pl, tomasz.blaszczyk@pcz.pl

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One of the important directions in the development of fractional calculus and its applications is the formulation of non-local models for multidimensional problems. Non-local effects and models based on fractional calculus have been widely investigated in one-dimensional settings, where they provide a natural tool for describing memory effects and interactions extending beyond the classical local approach. Their extension to two-dimensional problems is still less common in the literature, mainly due to the increased analytical and computational complexity. This difficulty becomes particularly relevant in continuum mechanics, where the formulation of plane stress or plane strain problems requires the construction of fractional operators acting consistently in more than one spatial direction.

In this work, we consider a two-dimensional fractional model of elasticity under the plane stress assumption [1]. The non-local character of the model is introduced by means of compositions of fractional derivatives with a fixed memory length. In contrast to fractional operators acting over the whole analyzed domain, the fixed-memory approach restricts the interaction range to a finite neighbourhood of each point. The use of composed left- and right-sided Caputo derivatives makes it possible to construct fractional counterparts of second-order and mixed differential operators appearing in the plane stress equations. This provides a flexible way to describe spatial memory effects in both coordinate directions.

Let us start with the left- and right-sided Caputo derivatives of order $\alpha \in (0,1)$ with fixed memory length $L > 0$, defined by [2]

$$ {}_{x-L}^{\;\;C}D_x^{\alpha} f(x) = \frac{1}{\Gamma(1-\alpha)} \int_{x-L}^{x} \frac{f'(s)}{(x-s)^{\alpha}} ds, \tag{1} $$

$$ {}_{x}^{C}D_{x+L}^{\alpha} f(x) = -\frac{1}{\Gamma(1-\alpha)} \int_{x}^{x+L} \frac{f'(s)}{(s-x)^{\alpha}} ds, \tag{2} $$

where Γ denotes the Gamma function [3]. In the proposed formulation, second-order fractional operators are constructed as symmetric compositions of the left- and right-sided Caputo derivatives. For example, in the x - direction we introduce the operator [4]

$$\mathcal{L}_x f(x,y) = \frac{1}{2}\left({}_x^C D_{x+L_x}^{\alpha}\, {}_{x-L_x}^{C} D_x^{\alpha} + {}_{x-L_x}^{C} D_x^{\alpha}\, {}_x^C D_{x+L_x}^{\alpha}\right) f(x,y) \tag{3}$$

where $L_x > 0$ denotes the fixed memory length in the x - direction. Analogous operators are introduced in the y - direction and for mixed derivatives.

Additionally, this work focuses on the numerical approximation of a 2D plane stress problem based on composed Caputo operators with fixed memory length.

References

[1] Sumelka W., Szajek K., Lodygowski T., Plane strain and plane stress elasticity under fractional continuum mechanics, Archive of Applied Mechanics 2014, 85(9).

[2] Blaszczyk T., Bekus K., Szajek K., Sumelka W., On numerical approximation of the Riesz--Caputo operator with the fixed/short memory length, Journal of King Saud University Science 2021, 33(1), 101220.

[3] Kilbas A.A., Srivastava H.M., Trujillo J.J., Theory and Applications of Fractional Differential Equations, Elsevier Science B.V., Amsterdam 2006.

[4] Klimek M., Blaszczyk T., Exact and approximate solutions of a fractional diffusion problem with fixed space memory length, International Journal of Applied Mathematics and Computer Science 2025, 35(2), 311-328.