MATHEMATICAL AND NUMERICAL ANALYSIS OF A FRACTIONAL DIABETES MODEL WITH SINGULAR AND NON-SINGULAR OPERATORS

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Diabetes mellitus is a chronic disease whose development is dependent on several interacting biological processes. Because of this complexity, mathematical models are often used to better understand the disease's evolution over time. In this paper, we propose a fractional-order compartmental model for diabetes that incorporates both singular and non-singular fractional operators. The primary purpose of using these two operators is to examine how different types of memory effect influence the system dynamics. The mathematical properties of the proposed model are first investigated. We establish the existence and uniqueness of solutions and show that the solutions remain positive and bounded for all relevant parameter values. The stability of the equilibrium states is then analyzed and the impact of the fractional orders on the long-term behavior of the model is discussed. To support the analytical results, numerical simulations are performed. These simulations illustrate how the choice of singular or non-singular operators affects the evolution of the diabetic population. The comparison shows that fractional-order models offer greater flexibility than standard integer-order derivatives and can better describe diabetes dynamics. The results of this study contribute to a better mathematical understanding of diabetes modeling and may serve as a foundation for further theoretical investigations.

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