MEASURES OF NONCOMPACTNESS AND THE SCHAUDER FIXED POINT THEOREM

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The concept of a measure of noncompactness was introduced by K. Kuratowski in 1930. In 1955, using Kuratowski's measure of noncompactness, G. Darbo generalized the classical Schauder fixed point theorem to the case of sets that are not compact. The concept of the Kuratowski measure of noncompactness was modified in 1957 by I. T. Gohberg, L. S. Goldenstein, and A. S. Markus, who introduced the so-called Hausdorff measure of noncompactness. In 1972, V. I. Istrăţescu defined another interesting measure of noncompactness. All these measures have the same fundamental properties, which are essential for applications (e.g. in the theory of integral equations). This fact suggested the idea of an axiomatic approach to defining measures of noncompactness. The first who went this way was B. N. Sadovskii. In the literature, one can find three different ways of axiomatically introducing measures of noncompactness. In this talk, we accept the one that was introduced in 1980 by J. Banaś and K. Goebel [1].

In addition to measures of noncompactness (sometimes referred to as strong measures of noncompactness), one also considers measures of weak noncompactness. This notion was introduced by F. S. De Blasi in 1977. Moreover, an analogue of the already mentioned Darbo theorem, using De Blasi's measure of weak noncompactness, was proved by De Blasi in the case of separable spaces and by G. Emmanuele for an arbitrary Banach space. An axiomatic approach to the notion of a measure of weak noncompactness—which is very similar to the approach associated with the notion of a measure of noncompactness in the strong sense—was proposed in 1988 by J. Banaś and J. Rivero [2].

In recent years, research has also been conducted on measures of noncompactness and their applications in Fréchet spaces. However, the axiomatic framework of such measures differs slightly from that of measures of noncompactness in Banach spaces.

In this talk we restrict our attention to strong measures of noncompactness in Banach spaces, and present several applications of these measures to generalize the Schauder Fixed Point Theorem [3].

References

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