NEMYTSKIJ COMPOSITION OPERATOR IN SOME FUNCTION SPACES

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An operator H acting between two classes of functions $X(I)$ and $Y(I)$ defined on a compact real interval I given by $H(f)(x):=h(x,f(x),)$ for some function $h:I\times R\to R$, is said to be a composition (Nemytskij or superposition) operator.

The goal of the presentation is to state the condition for the function space $X(I)$ under which the generator of each Nemytskij composition operator mapping $X(I)$ into the class of continuous functions $C(I)$ is continuous. It is worth noting that the continuity of h, contrary to expectation, is not obvious. It happens that the generating function is not adequate regular to the classes of the functions on which the corresponding operator is defined. The Matkowski example shows, that a discontinuous function can generate a Nemytskij operator mapping the space of continuously differentiable functions into itself. Moreover, we will give some consequences concerning the Nemytskij operators acting between the classes of continuous functions of bounded variation in the sense of Waterman and in the sense of Schramm.

References

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