APPLICATION OF VOLTERRA-STIELTJES INTEGRAL EQUATIONS IN DESCRIBING NEURON DYNAMICS UNDER AN ELECTROMAGNETIC FIELD

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In this talk, I will present results on the solvability over a bounded interval of a system of nonlinear integral equations that generalize a fractional-order neuron model formulated using Caputo fractional derivatives. The results presented are contained in [2]. The starting point of our analysis is a correction and refinement of results established in [1]. It turns out that the previous formulation was not fully accurate, as the multidimensional system comprising membrane voltage, recovery current, charge, and magnetic flux, was incorrectly reduced to a one-dimensional integral equation.

Specifically, we propose an approach based on replacing the considered neuron models with a system of nonlinear Volterra–Stieltjes integral equations. This formulation allows us to apply tools of nonlinear functional analysis, such as fixed-point theorems in appropriate Banach spaces, to obtain transparent and easily applicable existence results. The established criteria significantly extend and refine the solvability theory for this class of generalized fractional neuron models, providing a robust mathematical foundation for further research in this area.

References

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