INTERNAL HEATING OF A HOMOGENEOUS SPHERE – FRACTIONAL DUAL-PHASE-LAG MODEL

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Keywords: **dual-phase-lag model, heat transfer, time-fractional derivative, Laplace transform**

The subject of this paper is a dual-phase-lag heat conduction model for a homogeneous sphere. In our research, we assume that the sphere is heated by an internal heat source, but no heat exchange with the surroundings occurs on the sphere's surface [1]. The starting point for our considerations is the equation:

$$\rho C_p\left(\frac{\partial^\beta}{\partial t^\beta}T(r,t)+\tau_q\frac{\partial^\alpha}{\partial t^\alpha}\frac{\partial^\beta}{\partial t^\beta}T(r,t)\right)=\nabla\cdot\left(k\nabla T(r,t)\right)+\tau_T\frac{\partial^\alpha}{\partial t^\alpha}\nabla\cdot\left(k\nabla T(r,t)\right)+g(r,t)+\tau_q\frac{\partial^\alpha}{\partial t^\alpha}g(r,t) \tag{1}$$

with appropriate boundary and initial conditions, the detailed derivation of which can be found in [2]. In the Equation (1), ρ is the density of the material; C_p is a specific heat of the medium; τ_q and τ_T are the thermal relaxation and thermalization times, respectively; k is the thermal conductivity of the material; r is the radial coordinate of the spherical region; t is the time; ∇, $\nabla\cdot$ are the gradient and divergence operators, and T is the temperature. Moreover, the function $g(r,t)=Q_1 H\left(\frac{R}{5}-r\right)$ is an internal heat source, where Q_1 is a constant and $H(\ )$ is the Heaviside step function.

The solution to the problem was derived analytically, but the inverse Laplace transform, which appears in the final solution, was determined numerically due to its complexity. More specifically, the Stefhest algorithm was used. The final part of the paper consists of two numerical examples that investigate the influence of fractional time derivatives on the temperature distribution in the sphere under consideration.

References

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