ACOUSTIC WAVE PROPAGATION IN A QUASI–ONE–DIMENSIONAL PHONONIC CRYSTAL WITH MECHANICALLY PHASE–SHIFTED DEFECTS

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In this study, tunable defects were introduced into a quasi–one–dimensional phononic structure composed of tube segments with different cross–sectional areas. The relative phase shift between the defects was controlled mechanically. The analysis was carried out using two numerical approaches: the Transfer Matrix Method [1]–[3], used to determine the frequency–dependent transmission spectrum, and the Finite–Difference Time–Domain method [4], [5], used to study transient wave propagation inside the structure. The results show that both the defect cross–sectional area and the relative phase shift between the defects significantly affect the positions of transmission bands, the width of band gaps, and the spatial distribution of acoustic wave amplitudes. The proposed configuration enables adaptive control of acoustic wave transmission in phononic structures.

The transmission T is determined on the basis of the characteristic matrix $\boldsymbol{M}$ consisting of the propagation matrix P and the transmission matrix Φ.

$$T = |\boldsymbol{M}_{11}|^{-2} \tag{1}$$

$$\boldsymbol{M} = \Phi_{in,1}\left[\prod_{i=1}^{N-1} P_i \Phi_{i,i+1}\right] P_N \Phi_{N,out} \tag{2}$$

$$P_i = \begin{bmatrix} e^{jk_i d_i} & 0 \\ 0 & e^{-jk_i d_i} \end{bmatrix}, k_i = \frac{\omega}{c_i} = \frac{2\pi f}{c_i} \tag{3}$$

$$\Phi_{i,i+1} = \frac{1}{t_{i,i+1}} \begin{bmatrix} 1 & r_{i,i+1} \\ r_{i,i+1} & 1 \end{bmatrix}, t_{i,i+1} = \frac{2Z_{i+1}}{Z_{i+1}+Z_i}, r_{i,i+1} = \frac{Z_{i+1}-Z_i}{Z_{i+1}+Z_i}, Z_i = c_i \varrho_i \tag{4}$$

In the finite difference time domain algorithm (FDTD), the propagation of a mechanical wave is described by a system of differential equations, which consists of the continuity equation and the Euler equation, respectively, as

$$\frac{1}{\rho c^2}\frac{\partial p(\vec{x},t)}{\partial t} = -\nabla \cdot \vec{v}(\vec{x},t);\ \rho\frac{\partial \vec{v}(\vec{x},t)}{\partial t} = -\nabla p(\vec{x},t) \tag{5}$$

where $p(\vec{x},t)$ is the pressure field in Cartesian space $\vec{x}$ and time t, $\vec{v}(\vec{x},t)$ is the vector velocity field, and ρ and c are the density and phase velocity of the mechanical wave propagating through the material, respectively.

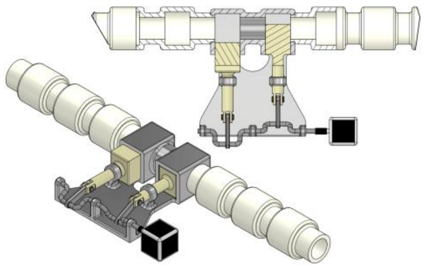

Fig. 1. Phonic crystal with phase–shifted defects and its cross-section.

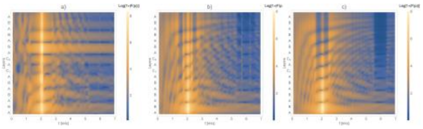

Fig. 2. Spectral distribution of a mechanical wave propagating inside a phononic structure.

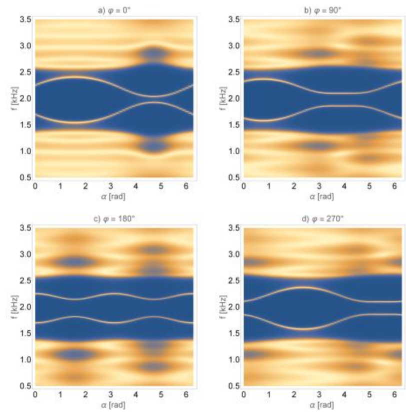

Fig. 3. Influence of the rotation angle α on the transmission of the analyzed structure for different phase shifts φ of defects.

Using TMM and FDTD methods, it was confirmed that changing the defect sizes (C_1, C_2) allows for effective shifting of the transmission bands and modification of the bandgap widths. A particularly pronounced effect was observed in the bandgap region around 2 kHz, where appropriately selected defect geometry enabled both near–complete wave attenuation and local resonance, leading to amplitude enhancement.

Introducing a controlled phase difference φ between defects further enhances the adaptive tuning capabilities of the structure. The greatest stability of the transmission characteristic was achieved with the antiphase φ = 180°, while the greatest changes in the positions and amplitudes of the transmission peaks were observed at φ = 0° as a function of the rotation angle α.

References

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